CARTAN ON COMPLEX PROJECTIVE GEOMETRY

Leçons de Géométrie Projective Complexe, par E. Cartan. Paris, Gauthier-Villars, 1931. vii+325 pp.

This volume has been prepared from the notes of M. F. Marty on a course of lectures given by Cartan at the Sorbonne during the winter of 1929–1930. As Cartan is one of the foremost geometricians of our time, one may, of course, expect that exposition and content of the treatise bear the marks of a master in his field. This is indeed the case, so that one may state without exaggeration that the book under review contains the most comprehensive and scientific treatment of complex geometry in existence.

It is hardly necessary to emphasize the importance of geometries in a complex domain. This is evident when we consider for example the transformation

$$z' = \frac{az + b}{cz + d}\cdot$$

When a, b, c, d are real constants and z', z real variables, we are concerned simply with real projective geometry on a line, with its relatively simple content. But let the constants and variables involved be chosen from the complex domain, so that we are now concerned with the projective geometry of the complex line. Everybody realises how enormously the field has been enlarged and enriched. We need to think of the interpretation by circular transformations in the complex plane only, and its important applications in various mathematical fields, to realize the utility of extending geometric investigation into the complex domain.

The same relative importance attaches to the extension to higher complex spaces as carried out in a masterly fashion by Cartan. Assuming all quantities chosen from the complex domain, one may say that the content of projective geometry in the complex domain is formed by those properties of geometric forms or varieties which are invariant under the projective transformation or collineation (*homographie*)

$$\begin{aligned} x_1' &= a_{11}x_1 + \cdots + a_{1n+1}x_{n+1}, \\ x_2' &= a_{21}x_1 + \cdots + a_{2n+1}x_{n+1}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ x_{n+1}' &= a_{n+11}x_1 + \cdots + a_{n+1n+1}x_{n+1}, \end{aligned}$$

or the projective anti-collineation (*antihomographie*)

$$\begin{aligned} x_1' &= a_{11}\bar{x}_1 + \cdots + a_{1n+1}\bar{x}_{n+1}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ x_{n+1}' &= a_{n+1\,1}\bar{x}_1 + \cdots + a_{n+1n+1}\bar{x}_{n+1}, \end{aligned}$$

in which $\bar{x}_i$ signifies the conjugate of x_i, and $|a_{ik}| \neq 0$.