ANALYTIC STUDY OF RATIONAL QUINTIC SURFACES HAVING NO MULTIPLE CURVES

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1. *Introduction.* The purpose of this paper is to derive the equations of certain of the rational quintic surfaces without multiple curves discussed synthetically by Montesano.* The equations of the surfaces are found by applying Cremona transformations to certain well known rational surfaces of order three or four.

2. *Surface of Order Five with Four Triple Points.* This surface is the transform by the cubic transformation T_{tet}† of a general cubic surface ϕ_3 through the vertices of the tetrahedron. The equation of the surface is

$$\begin{aligned}\phi_5 \equiv y_1^2\,[y_2^2 u + y_3^2 u' + y_4^2 u'' + A y_2 y_3 y_4] \\ + y_1[y_4^2\phi_2 + y_2 y_3(B y_3 y_4 + y_2 u''')] \\ + y_2 y_3 y_4[C y_3 y_4 + D y_2 y_4 + E y_2 y_3] = 0,\end{aligned}$$

where u, u', u'', u''' are linear in (y_3, y_4), (y_2, y_4), (y_2, y_3), (y_3, y_4), respectively, and ϕ_2 is quadratic in (y_2, y_3), and where A, B, C, D and E are constants. The points whose coordinates are (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1) are triple points, with non-composite tangent cones at each of the points.

3. *The Surface ϕ_5 with Three Ordinary Triple Points and a Tacnode.* The surface ϕ_5 is the transform by T_{tet} of a quartic surface with a double conic passing through three of the vertices of the tetrahedron and having the fourth vertex at a general point of the surface. The section of ϕ_5 by a plane through the tacnode and two triple points is a straight line and a pair of conics passing through these points.

The equation of ϕ_4 with a double conic is

$$\phi_4 \equiv [\,\textstyle\sum a_i x_j x_k]^2 - 4x_4^2\,[\psi_2 + x_4\psi_1] = 0,$$
$$(i, j, k = 1, 2, 3, 4, \cdots, i \neq j \neq k),$$

* Montesano, Napoli Rendiconti, (3), vol. 7 (1901), pp. 67–106.

† Hudson, *Cremona Transformations in Plane and Space*, Cambridge University Press, 1927, pp. 301–303.