NOTE ON AN APPLICATION OF METRIC GEOMETRY TO DETERMINANTS

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1. *Introduction.* This note refers to a previous paper* published in this Bulletin. On page 754 of that paper, the following theorem is stated.

THEOREM. *If the symmetric determinant*

$$D = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & r_{12} & r_{13} & r_{14} \\ 1 & r_{21} & 0 & r_{23} & r_{24} \\ 1 & r_{31} & r_{32} & 0 & r_{34} \\ 1 & r_{41} & r_{42} & r_{43} & 0 \end{vmatrix}, \quad (r_{ij} = r_{ji}),$$

with $r_{ij}>0$, $(i, j=1, 2, 3, 4)$, $i\neq j$, *is different from zero, and the complementary minors of four of the elements in the principal diagonal vanish, then the complementary minor of the remaining element does not vanish.*

In order to prove this theorem, two cases, A and B, were considered. In Case A it was supposed that all four of the bordered complementary minors vanished. It was then shown that the fifth complementary minor (the unbordered minor that is the complementary minor of the element appearing in the first row and first column) did not vanish.

In Case B it was assumed that three of the four bordered minors and the unbordered minor, denoted by $\mathcal{E}(p_1, p_2, p_3, p_4)$, vanished. It was stated that Case B was exhausted by a study of two sub-cases, both of which were examined and found to contradict the hypotheses stated for Case B. It was concluded, then, that this case could not exist, and from this conclusion four interesting corollaries were stated. The principal interest of the paper seems to the writer to lie in these corollaries.

A communication received from W. V. Parker called my attention to the fact that the hypotheses explicitly made upon

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* This Bulletin, vol. 37 (1931), pp. 752–758.