NOTE ON THE DIOPHANTINE EQUATION $ax^2 + by^2 + cz^2 + dt^2 = 0$

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Suppose that the constants a, b, c, d are integers none of which is zero, and that

I. *a, b, c, d have no squared factors;*

II. *no three of a, b, c, d have a common factor, that is,* $[a, b, c] = 1$, *etc.*

Then, if we suppose the restrictions I, II apply throughout the paper, the equation

$$ax^2 + by^2 + cz^2 + dt^2 = 0 \tag{1}$$

has integer solutions not all zero, if, and only if,

III. *a, b, c, d are not all of the same sign,*

IV. *every odd prime factor p_{ab} of $[a, b]$ for which*

$$\left(\frac{-cd}{p_{ab}}\right) = -1, \tag{2}$$

must also satisfy

$$\left(\frac{-ab/p_{ab}^2}{p_{ab}}\right) = 1, \tag{3}$$

together with five corresponding conditions derived by permuting the letters. The symbol is that of quadratic residuacity.

V. (1) *Either* $abcd \equiv 2, 3, 5, 6, 7 \pmod 8$,

V. (2) *or* $abcd \equiv 1 \pmod 8$, $a+b+c+d \equiv 0 \pmod 8$;

these mean that we must exclude

$$a \equiv b \pmod 8, \qquad c \equiv d \pmod 8, \qquad a \equiv c \pmod 4,$$

and the corresponding sets derived by permuting the letters;

V. (3) *or two of a, b, c, d are even, say a, b, and we must have*

V. (3.1) *either* $\frac{1}{4}abcd \equiv 3, 5, 7 \pmod 8$,

V. (3.2) *or* $\frac{1}{4}abcd \equiv 1 \pmod 8$, and $\frac{1}{2}a + \frac{1}{2}b + c + d \equiv (c^2d^2 - 1)/2 \pmod 8$; these mean that we must exclude the sets

$$\frac{1}{2}a \equiv \frac{3}{2}b \pmod 8, \quad c \equiv 3d \pmod 8,$$