NEW DIOPHANTINE AUTOMORPHISMS*

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1. *Introduction.* In this note we consider the construction of an infinite set of homogeneous polynomials f such that

$$f(Y_1, \cdots, Y_n) = f^h(y_1, \cdots, y_n), \tag{1}$$

where

$$Y_i = Y_i(y_1, \cdots, y_n), \qquad (i = 1, \cdots, n), \tag{2}$$

are homogeneous of degree h. In that case, f is said to admit of a diophantine automorphism.‡ The construction depends on a very simple principle connected with invariant theory.

Let ϕ be a binary form of degree δ:

$$\phi = a_x^\delta = a_0x_1^\delta + \binom{\delta}{1}a_1x_1^{\delta-1}x_2 + \cdots + \binom{\delta}{\delta}a_\delta x_2^\delta;$$

let q be a quadratic covariant of ϕ of degree ρ; then ψ, the discriminant of q, is an invariant of the ground-form ϕ of degree 2ρ. By the property of invariance, if under the linear transformation

$$x_1 = \alpha_{11}X_1 + \alpha_{12}X_2, \quad x_2 = \alpha_{21}X_1 + \alpha_{22}X_2, \tag{3}$$

of determinant $\Delta = \alpha_{11}\alpha_{22} - \alpha_{21}\alpha_{12}$, the form ϕ becomes

$$\Phi = A_X^\delta = A_0X_1^\delta + \binom{\delta}{1}A_1X_1^{\delta-1}X_2 + \cdots,$$

and ψ becomes $\Psi = \psi(A)$, then

$$\psi(A) = \Delta^{\delta\rho}\psi(a). \tag{4}$$

Now putting $q = q_{11}x_1^2 + 2q_{12}x_1x_2 + q_{22}x_2^2$, we take in (3)

$$\alpha_{11} = q_{12}, \quad \alpha_{12} = q_{22}, \quad \alpha_{21} = -q_{11}, \quad \alpha_{22} = -q_{12}.$$

Evidently (4) becomes

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‡ E. T. Bell, this Bulletin, vol. 33 (1927), pp. 71–80.