SOLUTIONS OF BOUNDED VARIATION OF THE FREDHOLM-STIELTJES INTEGRAL EQUATION*

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The purpose of this note is to give conditions under which the Stieltjes integral equation

$$\phi(x) = f(x) + \lambda \int_a^b K(x, y) d\phi(y) \tag{1}$$

has a solution $\phi(x)$ of bounded variation.

In the first theorem conditions on $f(x)$ and $K(x, y)$ are given under which the method of successive substitutions yields a solution of bounded variation for a limited range of values of λ.

With further restrictions on $K(x, y)$, it is shown in the second theorem that the Fredholm method applies and the solution of bounded variation thus obtained is valid for all values of λ except for the characteristic values.

Finally, an example is given to show that the more restrictive conditions on $K(x, y)$ given in the second theorem are not sufficient to make the problem a special case of that treated by Riesz.†

THEOREM 1. *If*

(a) *$f(x)$ is of bounded variation, $a \leqq x \leqq b$,*

(b) *$K(x, y)$, defined and bounded on $R(a \leqq x \leqq b,\ a \leqq y \leqq b)$, is continuous in y for each x and has a total variation in x for each y, $T_K(y)$, which is a bounded function of y having the least upper bound T_K and*

$$|\lambda| < 1/T_K, \tag{c}$$

then the function $\bar{\phi}(x)$ defined by the series

$$\bar{\phi}(x) = f(x) + \lambda \int_a^b K(x, y_1) df(y_1) + \lambda^2 \int_a^b K(x, y_1) d \int_a^b K(y_1, y_2) df(y_2) + \cdots \tag{2}$$

is the unique solution of bounded variation of integral equation (1).

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† F. Riesz, *Über lineare Funktionalgleichungen*, Acta Mathematica, vol. 41 (1918), pp. 71–98.