ON A MATRIX DIFFERENTIAL OPERATOR*

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1. *Introduction.* H. W. Turnbull† has defined a matrix differential operator and has given several theorems concerning its properties. The first of these theorems is proved by induction, and for the fourth, use is made of a lemma concerning the principal minors of a determinant for which he gives a long and intricate proof. It is the purpose of this note to give a simple and direct proof of this lemma and of the fourth theorem, and to indicate how each of the first three theorems proved by Turnbull may be obtained directly. In the last paragraph will be given, in addition, the results of applying the operator successively to the coefficients of the characteristic equation of a matrix.

2. *The Matrix Differential Operator.* Let

$$E = \begin{Vmatrix} E_1^1 & E_2 & \cdots & E_n^1 \\ E_1^2 & E_2^2 & \cdots & E_n^2 \\ \cdot & \cdot & \cdot & \cdot \\ E_1^n & E_2^n & \cdots & E_n^n \end{Vmatrix}$$

be an n-rowed square matrix whose n^2 elements are treated as independent variables. This matrix is characterized by a typical element, and we shall write $E = E_s{}^r$, where r denotes the row and s the column. In this way if we have another matrix $F = F_s{}^r$, we shall write,‡ from the law of multiplication of matrices,

$$EF = E_\alpha{}^r F_s^\alpha .$$

We observe that

$$EF = E_\alpha{}^r F_s^\alpha = F_s^\alpha E_\alpha{}^r ,$$

whereas

$$FE = F_\alpha{}^r E_s^\alpha = E_s^\alpha F_\alpha{}^r \neq EF.$$

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† H. W. Turnbull, *On differentiating a matrix,* Proceedings of the Edinburgh Mathematical Society, (2), vol. 1, Part 2 (1928), pp. 111–128.

‡ Throughout this paper repeated Greek suffixes will denote summation from 1 to n.