Let B denote the collection of all the classes $C_{(r,x)}$. Then B is an algebra. The subset A' of B consisting of all the classes $C_{(1,x)}$ of B is isomorphic with A and is identical with B if, and only if, for each element x of A and each positive rational integer m, A contains an element x/m such that $m(x/m)=x$.

If $C_{(r,x)}$ is an element of B and m is a positive rational integer, then $C_{(r/m,x)}$ is an element of B and $mC_{(r/m,x)}=C_{(r,x)}$.

Multiplication in B is associative, commutative, distributive, respectively, if the same is true for A.

PRINCETON UNIVERSITY

ON THE RANK OF THE PRODUCT OF CERTAIN SQUARE MATRICES*

BY W. O. MENGE

1. *Introduction.* This paper presents several theorems which were found during an investigation conducted by the author into the structure of matrices which transform given matrices into their so-called classic and rational canonical forms.† When the elementary divisors of a given matrix are known, these theorems completely determine the rank of a product of matrices of the form

$$\prod_{i=1}^{\omega}(A - \lambda_i I)^{k_i}.$$

An interesting proof of the Hamilton-Cayley theorem and a determination of the equation of minimum degree satisfied by a matrix are obtained from this point of view.

2. *Invariant Factors.* Consider the square matrix $A=(a_{ij})$ of order n with constant elements. If the n-rowed identity matrix be denoted by I, the characteristic matrix $(A-\lambda I)$ is defined as the matrix obtained by subtracting the variable λ from each principal diagonal element of A. The determinant, $D(\lambda)$, of the characteristic matrix $(A-\lambda I)$ is called the characteristic de-

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* Presented to the Society, December 30, 1930. The author wishes to acknowledge his appreciation to J. A. Nyswander, University of Michigan, for many helpful suggestions throughout the progress of the work.

† Dickson, *Modern Algebraic Theory*, Chap. 5.