CONCERNING ADJUNCTIONS TO ALGEBRAS

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In §3 of his paper, *Algebras which do not possess a finite basis*,† J. H. M. Wedderburn gives a set of postulates and definitions for an algebra. The question of the possibility of adjoining an identity to such an algebra arises. The purpose of the present paper is to show that this adjunction is always possible.

It may be seen from the arguments that Theorems 1 and 2 remain true if the term *algebra* be replaced by one implying a set of elements of which it is assumed only that it is an abelian group under addition and a semi-group under multiplication. The proof of Theorem 3 employs a distributive property, as will be indicated.

THEOREM 1.‡ *If A is an algebra, then there exists an algebra B which contains a proper, invariant subalgebra A' isomorphic§ with A and an element I not in A' such that, for every element b of B, the relation $Ib=bI=b$ holds.*

If x is an element of A and n is a positive rational integer, let $nx=xn$ denote the sum $x+x+\cdots+x$ (n summands) and let $(-n)x=x(-n)$ denote the same sum as $n(-x)$. Let $0\cdot x=x\cdot 0=0$.

Let B denote the set of all pairs (n, x) where n is a rational integer and x is an element of A. Suppose that $(n, x)+(m, y)=(n+m, x+y)$. Then B is an abelian group under addition. Let $(n, x)\ (m, y)=(nm, ny+mx+xy)$. Then B is an algebra and the subset A' of B consisting of all the elements of B of the form $(0, x)$ is isomorphic with A and is invariant in B. Let I denote the element $(1, 0)$ of B; $(1, 0)\ (n, x)=(n, x)\ (1, 0)=(n, x)$ for every n and x.

The multiplication in B is distributive, associative and distributive, commutative, respectively, provided the same is true for A.

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‡ For a proof of this theorem for an algebra with a finite basis, see, for example, L. E. Dickson, *Algebras and their Arithmetics*, 1923, p. 97.

§ In this paper isomorphism means *simple* isomorphism.