If the rank of the jacobian matrix is $n-2$, that is, if there exist two relations (and not more) between the f's which do not involve the x's, the hypersurfaces will all have the origin as a double point, and terms of the second degree of (10) tell us the nature of the singular point.

We note that equation (7) would be satisfied if every element vanished. This would lead to n^2 linear equations in the quantities f_k'. It is known that if the algebra is commutative and associative, these n^2 equations have a unique solution for all analytic functions of w. The derivative congruence would in this case degenerate into a congruence of points, the function being monogenic.

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ON FINITE AND INFINITE COMPLETELY MONOTONIC SEQUENCES*

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1. *Introduction.* As one of several important results concerning the problem of moments for a finite interval, Hausdorff† has proved the following theorem.

The problem of moments, which is to find a real function $\chi(t)$ *for* $0 \leqq t \leqq 1$, *such that*

$$(1) \qquad \mu_n = \int_0^1 t^n d\chi(t), \qquad (n = 0, 1, 2, \cdots),$$

in which the sequence $\mu_0, \mu_1, \mu_2, \cdots$ *is given in advance, has a monotonic non-decreasing solution, if and only if*

$$(2) \quad \Delta^l \mu_m = \sum_{\nu=0}^{l} (-1)^\nu \binom{l}{\nu} \mu_{m+\nu} \geqq 0, \qquad \text{for } l, m = 0, 1, 2, \cdots,$$

in which case the sequence is said to be completely monotonic.

The same problem (1) *has a solution* $\chi(t)$ *which is of bounded variation, if and only if*

* Presented to the Society, November 28, 1931.

† *Ueber das Momentenproblem für ein endliches Interval,* Mathematische Zeitschrift, vol. 16 (1923), pp. 220–248. We refer to §§1 and 2.