POLYGENIC FUNCTIONS OF HYPERCOMPLEX VARIABLES

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1. *Introduction.* The purpose of this paper is to generalize to hypercomplex variables some of the results obtained by Kasner in the papers recently published by him on polygenic functions of the complex variable z.* Kasner has applied the term *polygenic* to any function of the form

$$f(z) = \phi(x, y) + i\psi(x, y),$$

where ϕ and ψ are arbitrary functions (except for suitable continuity and differentiability assumptions) of the real variables x, y, the Cauchy-Riemann conditions not being assumed.

If the first derivative df/dz, which is denoted by $\gamma = \alpha + i\beta$, be plotted in the α, β plane, the locus of the points γ corresponding to a given point z is a circle

$$\left(\alpha - \frac{\phi_x + \psi_y}{2}\right)^2 + \left(\beta - \frac{-\phi_y + \psi_x}{2}\right)^2 = \left(\frac{\phi_x - \psi_y}{2}\right)^2 + \left(\frac{\phi_y + \psi_x}{2}\right)^2.$$

To the ∞^2 points of the z-plane correspond ∞^2 circles. This congruence of circles is called the *Kasner derivative congruence.* (In special cases where the function f is monogenic the circles all shrink to points.)

Hedrick, Ingold, and Westfall† developed the theory of non-analytic functions of a complex variable in a paper published in 1923 and E. R. Hedrick in a later paper‡ pointed out relations

* E. Kasner, *Theory of polygenic functions*, Science, vol. 66 (1927), pp. 581–582, and Proceedings of the National Academy of Sciences, vol. 14 (1928), pp. 75–82; *Note on the derivative circular congruence of a polygenic function*, this Bulletin, vol. 34 (1928), pp. 561–565.

† Hedrick, Ingold and Westfall, *Theory of non-analytic functions of a complex variable*, Journal de Mathématiques, (9), vol. 2 (1923), pp. 327–342.

‡ E. R. Hedrick, *On derivatives of non-analytic functions*, Proceedings of the National Academy of Sciences, vol. 14 (1928), pp. 649–654.