APOLARITY IN THE GALOIS FIELDS OF ORDER 2^n*

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Let us consider an m-ary quadratic in the Galois fields of order 2^n

$$f(x_1, x_2, \cdots, x_m) \equiv \sum a_{ij}x_ix_j = 0, \tag{1}$$

where

$$i, j = 1, 2, \cdots, m; j \geqq i; a_{ji} = 0 \text{ if } j \neq i.$$

If m is even, the discriminant of (1) is†

$$\Delta \equiv \begin{vmatrix} 0 & a_{12} & a_{13} & \cdots & a_{1m} \\ a_{12} & 0 & a_{23} & \cdots & a_{2m} \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ a_{1m} & a_{2m} & a_{3m} & \cdots & 0 \end{vmatrix}. \tag{2}$$

If m is odd, the discriminant of (1) is*†

$$\Delta \equiv \frac{1}{2} \begin{vmatrix} 2a_{11} & a_{12} & \cdots & a_{1m} \\ a_{12} & 2a_{22} & \cdots & a_{2m} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{1m} & a_{2m} & \cdots & 2a_{mm} \end{vmatrix}. \tag{3}$$

We note that in the expansion of (2) we shall have terms like $2a_{12}a_{23}a_{34} \cdots a_{1m} \equiv 0$ modulo 2. Hence (2), when expanded, is of the form

$$a_{12}^2a_{34}^2a_{56}^2 \cdots a_{m-1m}^2 + a_{13}^2a_{24}^2 \cdots + \cdots$$
$$+ (a_{12}a_{34}a_{56} \cdots a_{m-1m} + a_{13}a_{24} \cdots + \cdots)^2.$$

Let us consider a pencil of m-ary quadratics

$$\sum(\lambda b_{ij} + \mu a_{ij})x_ix_j = 0, \tag{4}$$

with b_{ij} and a_{ij} like a_{ij} in (1).

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* Presented to the Society, December 28, 1931.

† See A. D. Campbell, *The discriminant of the m-ary quadratic in the Galois fields of order* 2^n, Annals of Mathematics, (2), vol. 29 (1928), No. 3, pp. 395–398.