ON THE APPLICATION OF A THETA FORMULA TO REPRESENTATION IN BINARY QUADRATIC FORMS*

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In his remarkable 1885 dissertation,‡ *Applications of the Theory of Elliptic Functions to the Theory of Numbers*, Nazimoff uses the formula (1) to derive the number of representations in x^2+3y^2 and x^2+5y^2. Some of his theorems are not quite correct. He does not derive the series representing $\sum\sum q^{x^2+7y^2}$ since "the derivation is long and not difficult." He states that the method has only limited application, since generally in (4) the A's are functions of k.

It is the purpose of this paper to give the essential details for x^2+7y^2, and to prove that the only cases ax^2+by^2, ab odd, which are actually solvable by Nazimoff's method are $a=1$, $b=1$, 3, 5, 7. The final result for x^2+7y^2 is of course far from new.

At the time this paper was written the writer intended to examine the products of series involved in other cases than $a=1$, $b=1$, 3, 5, 7, in the hope of obtaining information on $N(n=x^2+11y^2)$ or other cases of several classes in a genus. But since then an arithmetic method has been discovered of finding a simple formula for the number of representations in any positive, binary quadratic form. It is possible, after the details of this theory have been worked out, that there may be applications to elliptic and modular functions.

The formula which Nazimoff uses is

$$\sum_{x,y} q^{ax^2+by^2} = (ab)^{-1/2}\phi(2K/(ab)), \tag{1}$$

where x, y run through all integers, a and b are given positive, odd, relative-prime integers, and

$$\phi(u) = \frac{2K}{\pi} \prod_{l=1}^{(a-1)/2} t(lbu) \prod_{h=1}^{(b-1)/2} t(hau). \tag{2}$$

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‡ Translation by Arnold E. Ross, pp. 5–12.