REPRESENTATION OF A GROUP AS A TRANSITIVE PERMUTATION GROUP

BY G. A. MILLER

Let G be any group of finite order g and let H be any subgroup of order h contained in G. If the operators of G are separated into right or into left augmented co-sets with respect to H and if these $g/h=n$ co-sets are then multiplied successively on the right or on the left respectively by the various operators of G, they will be permuted as units according to a transitive permutation group T which is simply isomorphic with the quotient group of G with respect to the largest invariant subgroup of G which appears in H, if H is not itself invariant under G. If H is invariant under G, then T will be a regular group which is simply isomorphic with G/H. The case when H is non-invariant under G and does not involve any invariant subgroup of G besides the identity is especially important since T is then simply isomorphic with G, as was pointed out for right co-sets by W. Dyck in 1883.

If K is any subgroup of G which has operators in each of the co-sets of G with respect to H and if K_0 is the cross-cut of H and K, then K_0 may be invariant under K or it may involve an invariant subgroup under K. If one of these conditions is satisfied, H must involve a subgroup which is invariant under G and includes this invariant subgroup under K. This follows directly from the facts that this invariant subgroup is transformed into all of its conjugates under G by operators of H and that a complete set of conjugate subgroups always generates an invariant subgroup if it does not generate the entire group. We have then the following result.

THEOREM 1. *If a group G is separated into co-sets with respect to a subgroup H and if another subgroup K has operators in each of these co-sets, then the largest invariant subgroup under K which appears in the cross-cut of H and K is contained in an invariant subgroup of G which is found in H.*

In particular, when T is simply isomorphic with G, then the largest invariant subgroup under K which appears in the cross-cut of H and K is the identity.