A NOTE ON PRIMITIVE IDEMPOTENT ELEMENTS OF A TOTAL MATRIC ALGEBRA*

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We consider a total matric algebra M over a field F, whose general element is

$$u = \sum \alpha_{ij} e_{ij}, \qquad (i, j = 1, \cdots, n),$$

where $e_{ij}e_{lk} = e_{ik}$ if $j = l$, and $e_{ij}e_{lk} = 0$ for $j \neq l$.

THEOREM 1. *A necessary and sufficient condition that* $u = \Sigma \alpha_{ij} e_{ij}$ *be idempotent in* M *is*

$$(1) \qquad \sum_s \alpha_{ps}\alpha_{sq} = \alpha_{pq}, \qquad (p, q = 1, \cdots, n).$$

This is seen immediately on writing

$$u^2 = \sum_{p,q,s} \alpha_{ps}\alpha_{sq} e_{pq}$$

and comparing with

$$u = \sum_{p,q} \alpha_{pq} e_{pq}.$$

THEOREM 2. *A necessary and sufficient condition for an idempotent element* u *to be primitive in* M *is*

$$(2) \qquad \alpha_{pi}\alpha_{jq} = \alpha_{pq}\alpha_{ji}, \qquad (p, q, i, j = 1, \cdots, n).$$

For, let

$$u = \sum_{r,t} \alpha_{rt} e_{rt}$$

be a primitive indempotent element of M. Let $\alpha_{ji} \neq 0$. Then the element $ue_{ij}u/\alpha_{ji}$ is idempotent in uMu, since

$$\left(\frac{ue_{ij}u}{\alpha_{ji}}\right)^2 = \frac{u \cdot e_{ij}ue_{ij} \cdot u}{\alpha_{ji}^2} = \frac{u\alpha_{ji}e_{ij}u}{\alpha_{ji}^2} = \frac{ue_{ij}u}{\alpha_{ji}}.$$

Hence† we have $ue_{ij}u/\alpha_{ji} = u$, and $ue_{ij}u = \alpha_{ji}u$. Equating coef-

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† Dickson, *Algebras and their Arithmetics*, p. 55.