

has particularly interesting properties due to the fact that any orientation can be defined in terms of three, which is equal to the number of dimensions of the hypersurface. Many special hypersurfaces are also of considerable interest; among these may be mentioned hyperspheres, ellipsoids, and minimal hypersurfaces. This is only a small indication of the many things treated in this part of the book.

V. Part five, Invariants (120 pages). A book on this subject that makes any claim to completeness must necessarily discuss this subject. The treatment is based on Lie's continuous groups and covers the subject quite adequately.

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*The Logic of Discovery.* By R. D. Carmichael. Chicago, The Open Court Publishing Co., 1930. 274 pp.

Thanks to the analyses of both mathematicians and philosophers the science of logic has enjoyed greater development in the last seventy-five years than in the entire twenty odd centuries of its previous history. Up to 1850 it was identified with the formal logic of Aristotle. Since then five generalizations beyond Aristotle have occurred, each one of which has produced a new and more fertile system of logic. These systems are (1) the logic of classes initiated by Boole and Schröder, (2) the logic of relations associated with the names Peirce, Royce, Russell and others, (3) the logic of propositional functions formulated by Russell, (4) the logic of systems or doctrines and system or doctrinal functions, recently developed into an even higher generalization by Sheffer, and (5) the calculus of propositions first formulated by Russell and Whitehead in the *Principia Mathematica*, recently reinterpreted by Wittgenstein, and modified and perhaps even transcended by Hilbert, C. I. Lewis, Weyl, and others.

Carmichael's *Logic of Discovery* is based upon the fourth of these generalizations, the logic of systems or doctrines. Nothing new concerning this logic appears in his book. Its originality consists in its specification of the changes in our conception of mathematics, of reasoning, and of scientific method in the physical and social sciences, which the established concepts of this logic entail.

The well known fact that logical inference in a doctrinal function depends solely on the form is brought into the foreground. The key to this is really the notion of the variable and the discovery of the formal properties of relations which came into the logic of systems from the logic of propositional functions and relations. Carmichael's use of an experiment carried on by Veblen is most convincing on this point. The equivalence of postulate sets for the same system or system function is also indicated. These considerations enable Carmichael to reveal the relativity of the ordinary linguistic statements of a system, and the formal and purely rational side of scientific procedure.

His most original contribution concerns our conception of scientific method. It has been known for a long time by all acquainted with the historical evidence that Bacon's naive theory of induction has very little to do with actual scientific procedure, particularly as it occurs in the most mature sciences. Recently it has become the custom to conceive of all scientific work in terms of the method of hypothesis. But both conceptions have a common presupposition: Scientific statements are considered in terms of a few isolated propositions. The logic of systems and the part which it plays in mathematical physics forces a modifica-