Geometry of Four Dimensions. By A. R. Forsyth. Cambridge University Press, 1930. Two Volumes, Vol. I, 468 pp., Vol. II, 520 pp.

In the past few years several books on the geometry of four dimensions have appeared. They have been written, usually, with the aim of presenting the geometry necessary for the understanding of the theory of relativity. The present book has no such end in view. The author wishes simply to present a complete account of that geometry. The question then arises, why four dimensions and not n dimensions. We are told, in the preface, that there are two reasons for this; first, four dimensions is the simplest generalization of three dimensions and by treating these generalizations in minute detail much may be gained; second, there is a traditional importance to four dimensions.

One might naturally suppose that these two volumes would contain all that is known about the geometry of four dimensions, but this is not the case, as one more or less familiar with the subject soon finds out. The general plan of the work is excellent and it is written in the characteristic Forsyth style. The method used is that of Gauss and the notation is that of Cayley. The treatment is wholly analytical. The space is euclidean and in the preface we are told that curvature of a curve measures its departure from a euclidean straight line and that the complete curvature of a surface is a measure of its deviation from an euclidean plane. It seems to me that these statements need considerable interpretation since a fixed plane, for example, is euclidean or not according to the measure used. The only reason I can see for assuming euclidean space to be at the bottom of things is because it is the simplest known.

The work is divided into five main parts:

I. The first part (203 pages) is an account of the geometry of lines, planes and hyperplanes and the relations of one to another. I know of nowhere else that this material can be found in one place. A complete discussion of parallelism and perpendicularity of these elements and of the shortest distance and angles between them is given. A chapter on rotations in four dimensions is also included.

II. Part two (147 pages) is devoted to the theory of curves. The treatment is quite analogous to the theory of curves in ordinary 3-space but we have a new curvature, the second torsion or tilt, which complicates the theory to some extent. A chapter on curves in n-space is also included.

III. Part three (115 pages) is on surfaces in 4-space. While some parts of the theory of surfaces in 4-space are quite analogous to that of surfaces in 3-space there are other parts of it which are quite different. Geodesics are analogous, but properties of ordinary surfaces which depend upon the normal do not generalize quite so easily. Lines of curvature and asymptotic lines are examples. Gaussian curvature is the same; but in four dimensions, mean curvature is a vector. There are other properties which depend upon the fact that four is an even number and that the dimension of the surface is half this number. These topics are all adequately treated in this part of the book.

IV. Part four (381 pages) is on hypersurfaces. Geodesic lines, lines of curvature, and asymptotic lines are easy generalizations from ordinary surface theory. Curvature however is again quite different. The product of the principal radii is quite different from the product of the principal radii of curvature of a surface in 3-space. The Riemannian curvature of a hypersurface in 4-space