EDGE ON THEORY OF RULED SURFACES

The Theory of Ruled Surfaces. By W. L. Edge. Cambridge, University Press, 1931. ix+324 pages.

The appearance of the present volume, written by a competent author and published by a recognized house, furnishes further evidence of the general re-awakening of interest in algebraic geometry. The purpose is to describe the algebraic ruled surfaces of orders three to six, by means of the configuration of the double curves and of the developables of bitangent planes.

Only a limited knowledge of projective and analytic geometry is presupposed; a general introduction to the theory of correspondences, geometry on an algebraic curve, and the general properties of ruled surfaces is provided in the Introduction. The plan there is to state the essential theorems, frequently with illustrations, then to give either the reference to its source or to its demonstration in a well known book. The proofs in this preparatory chapter are not included. Two methods of procedure are followed; either one would be sufficient to accomplish the purpose, but when one becomes difficult to apply, the other may suggest new phases of attack. This double derivation provides a comprehensive check on the accuracy of the result.

The first method consists in mapping the lines of ordinary space S_3 on the points of a general quadric manifold M_4^2 of four dimensions and order two in S_5, as proposed by Klein. Ruled surfaces of S_3 are then represented by curves of the same order and same genus on M. The latter are classified according to the dimensionality of the space in which the curve lies; then, if in S_4, according to the number of their intersections with the generating planes of the two systems on M; then, within each category, by the number of actual multiple points. Thus, if C lies in the section of M by a general S_4, the ruled surface belongs to a non-special linear complex, and is properly self dual, that is, each generator is transformed into itself under polarity as to this complex. If the S_4 is tangent to M, the polarity does not exist, and the ruled surface has a rectilinear directrix, which may or may not also be a generator, according as the curve does or does not pass through the point of contact.

No sufficient criteria have been developed to account for all particular forms of the curve which give rise to ruled surfaces having tacnodal or oscnodal multiple curves. With the exception of those surfaces contained in a linear congruence with coincident directrices, no attempt is made to include such cases; indeed their existence is not even mentioned.

The other method is the projection of a normal ruled surface from a higher space, as given by Segre. This is entirely practicable for rational ruled surfaces and for others of low orders, but becomes very cumbersome for higher genera. In the extreme cases the determination of the normal surface reduces to the very problem it is proposed to solve. The methods of projection into S_3 suggest various correspondences by which the surfaces may be generated. Frequently these are not the easiest nor the most rational methods of generation. A chapter on developables includes the proof that all developables of orders less than 8 are