SPACES SATISFYING THE FIRST ENUMERABILITY AXIOM†

BY SELBY ROBINSON

1. *Introduction.* A neighborhood of a point p is a set of points to which p is interior, p being interior to a set V if p is a limit point of no subset of $C(V)$ and V contains p.‡ Using throughout the notation and concepts of Chittenden (loc. cit.), we define a topological space (P, K) as a collection P of points, and an undefined relation K giving for each subset E of P a unique set $K(E)=E'$ called the set of all limit points of E. Then $L(E)$ is the set of all limit points of all subsets of E; and (P, L) a space formed from (P, K) by taking as E' the set $L(E)$.

A space has property D of Hausdorff if for any pair of points there are disjoined sets to which the points are respectively interior; and is regular if, instead of for every pair of points, the property holds for every pair of disjoined sets provided each contains all its L points. Two families F and F' of neighborhoods of a point are equivalent if each set of F contains some set of F' and vice versa. For brevity we use $[U_p]$ consistently as a symbol for the family of all neighborhoods of the point p. The first enumerability axiom states that for each point p there is an enumerable family of neighborhoods equivalent to $[U_p]$. As our definition of a space V_ω, we adopt the one given by Fréchet in *Espaces Abstraits*, that is, a V space in which for each point p there is an enumerable decreasing family of neighborhoods whose product is p and which is equivalent to $[U_p]$. Fréchet had in a previous definition required in addition that a space V_ω be an L space.§ By the fourth property of Riesz,¶ we mean that

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‡ Fréchet, *Esquisse d'une théorie des ensembles abstraits*, Sir Asutosh Mookerjee's Commemoration volumes, Calcutta, 1922, vol. II, p. 362; Chittenden, Transactions of this Society, vol. 31 (1929), p. 296 and p. 293. By mistake the last clause was omitted from Chittenden's definition of interiority.

§ See *Les Espaces Abstraits*, Paris, 1928, p. 216; and Transactions of this Society, vol. 19 (1918), p. 56.

¶ *Espaces Abstraits*, pp. 209–10; D. McCoy, Tôhoku Mathematical Journal,