CRITERIA FOR THE SOLUTION OF A CERTAIN QUADRATIC DIOPHANTINE EQUATION

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1. *Introduction.* The interesting diophantine equation,

$$ax^2 + by^2 + cz^2 + du^2 = 0, \tag{1}$$

in which a, b, c, d are integers, all different from zero, and x, y, z, u are the unknowns, has already been treated in the literature. It is, however, desirable to have a complete treatment and definite statement of criteria for solvability directly applicable to a given equation.

In 1884 A. Meyer* stated, though somewhat obscurely, necessary and sufficient conditions for the solvability of equation (1), but his proof, as well as P. Bachmann's† treatment, is restricted to the case in which a, b, c, d are all odd integers. In 1930 an account of this equation was given by L. E. Dickson;‡ but, as he points out himself, his work is incomplete in the case in which exactly two of the coefficients are even integers and $abcd/4 \equiv 5 \pmod 8$. Recently a paper§ by L. J. Mordell has appeared in which a complete and independent derivation of conditions for solvability is given. His conditions, however, are not always directly applicable to a given equation: more explicitly, his condition II may not be satisfied by a given equation and yet the equation may possess a solution. According to the scheme there presented, restrictions are placed on the values of the unknowns, and it may happen that a set of diophantine equations have to be tested to determine whether the given one is solvable or not. This is best illustrated by an example.

His method is not directly applicable to the equation

$$110x^2 + 770y^2 - z^2 - u^2 = 0,$$

which is solvable for $x = 2$, $y = 1$, $z = 11$, $u = 33$). We enquire,

* Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich, vol. 29 (1884), pp. 209–222.

† *Zahlentheorie*, Part IV, *Die Arithmetik der Quadratischen Formen*, I, pp. 259–266.

‡ *Studies in the Theory of Numbers*, pp. 70–76.

§ Journal für Mathematik, vol. 164 (1931), pp. 40–49.