ON COMPLETE SYSTEMS UNDER CERTAIN FINITE GROUPS

BY C. W. STROM

1. *Introduction.* The theory of complete systems of invariants for rational functions that are unaltered under the substitutions of a finite group finds a close parallel in the theory of the binary n-ic.

It is well known that all the invariants and covariants of a binary n-ic can be expressed as rational functions of just n explicitly known forms such that the denominators are powers of the n-ic itself. In the parallel case Lagrange's theorem states that every rational function $F(x_1, \cdots, x_n)$ that is unaltered under the substitutions of a finite group G on $x_1, \cdots, x_n$ can be expressed as a rational function of the elementary symmetric polynomials $E_i, (i=1, \cdots, n)$, and any particular F that belongs to G. In these representations the denominator is a particular symmetric polynomial depending only on F.

If complete integrality is insisted upon, it is well known that the number of members in the complete system of the binary n-ic is finite although the exact number and the explicit forms cannot in general be determined by known means. In this paper we study the problem of obtaining irreducible sets of polynomials such that all polynomials that are invariant under the substitutions of a certain finite group can be expressed as rational, integral functions of the members of the irreducible set. In the case of the symmetric group $G_{n!}^n$ the answer is given by the fundamental theorem of symmetric functions. The results for the alternating group $G_{n!/2}^n$ and the identity group G_1 are also well known. We present solutions for the cyclic groups, the solvable groups, and the simple group G_{168}^7. In the cases of the cyclic groups and the solvable groups, the number of members in the irreducible sets is shown to be finite but the determination of the exact number is resolved essentially into a problem of partitions and can therefore not be expressed in general by known means.

If we regard $x_1, \cdots, x_n$ as coordinates in S_{n-1}, we may regard the group G as a collineation group and the results may be in-