ON A PROBLEM OF N. ARONSZAJN AND AN AXIOM OF R. L. MOORE*

BY LEO ZIPPIN†

In an interesting paper N. Aronszajn‡ raises the question whether every space belonging to a class of topologic spaces which he introduces§ is necessarily an absolute G_δ-set; that is, one which in every metric space M containing it may be expressed as the product of a countable set of sets open in M. In this, the first part of the paper gives an affirmative answer. The class of spaces $\mathfrak{R}$ is given, in effect, by the following axiom: a topologic space R belongs to the class $\mathfrak{R}$ if and only if there exists an infinite sequence of classes $\Pi_R{}^n(R)$ for $n=1, 2, \cdots$, such that

(1) for every natural number n, R is generated by $\Pi_R{}^n(R)$;

(2) with every sequence of sets $\{\overline{U}_n\}$, where $U_n \supset U_{n+1}$, there is associated a point p of space such that $p=\Pi_{i=1}^{\infty}\overline{U}_i$ and this sequence converges to p.

The necessary definitions are these: a class $\Pi_R(R)$ is any class of point sets of R which are open in R and whose sum covers R. Such a class generates R if, further, given any point p of R and any open set U containing p, then there is an element V of $\Pi_R(R)$ which contains p and is contained in U. A set of sets converges to a point if every open set containing the point contains all but a finite number of the given sets.

We suppose then that R is a space as defined above and is a subset of a metrizable space M. Then whatever metric be associated with M (preserving its topology) this is induced on R as a subspace (preserving *its* topology) and we may speak of the diameters of the elements of any of the generating classes. Let U be an element of one of these classes whose diameter is finite

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† National Research Fellow.

‡ N. Aronszajn, *Über die Bogenverknüpfung in topologischen Räumen*, Fundamenta Mathematicae, vol. 15 (1930), pp. 228–241.Compare the first paragraph of page 229.

§ Part of this paper is concerned with the relation of these spaces to spaces defined by R. L. Moore considerably earlier.