THE PROBABLE ERROR OF CERTAIN FUNCTIONS OF THE ERRORS MADE IN MEASUREMENTS*

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1. *Introduction.* This paper presents with proof and applications several theorems pertaining to the probable error of certain functions of the errors made in measurements. If the measurements or observations are multiplied by a constant b, the probable value of functions of the error of bm will differ from the probable value of these same functions of the error of m the measurement. This article shows that there are values of b such that the probable value of certain functions of the error of bm is less than the probable value of these functions of the error of m.

The probable value of certain functions of the mean are also treated. General frequency laws for the errors are used in the first theorems; these include the discrete, the continuous and a combination of the discrete and the continuous cases. Here Stieltjes integrals are used in the proofs. Special cases are mentioned.

The following theorems are proven by use of general frequency laws which come under the continuous case. The Gaussian law is treated as special cases to the theorems.

2. *Concerning the Square of the Error of the Measurement.*

THEOREM 1. *Let x be the error of the measurement m, d the expected value of x, and c the expected value of x^2. Then under any law of error, whose second moment with respect to the true value a exists, there exist values of the constant b such that the probable value of the square of the error of bm is less than the probable value of the square of the error of the measurement, provided*

$$ad - c \neq 0, \qquad a^2 - 2ad + c \neq 0.$$

Under these conditions b lies between

$$1 \quad \text{and} \quad 1 - \frac{2(c - ad)}{a^2 - 2ad + c}.$$

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