NOTE ON THE CONVERGENCE OF A SEQUENCE OF APPROXIMATING POLYNOMIALS*

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Various results have been published, by the present writer and others, with regard to the convergence of sequences of approximating polynomials defined by minimizing an integral of the form

$$\int_a^b \rho(x) \left| f(x) - P_n(x) \right|^m dx, \tag{1}$$

in which $f(x)$ is a function defined and continuous for $a \leqq x \leqq b$, and subject to such further hypotheses as the particular convergence proof in question demands, $\rho(x)$ is summable and non-negative over the interval, m is a given positive number >1, and $P_n(x)$ is a polynomial of the nth degree (this expression being understood throughout to mean a polynomial of the nth degree *at most*). The proof of convergence as n becomes infinite is most easily obtained if $\rho(x)$ has a positive lower bound, at least over an interval containing the point at which convergence is to be shown.† The author has briefly discussed for the corresponding trigonometric case the question of convergence at a point at which the weight function vanishes, under certain restrictive hypotheses as to the manner of vanishing.‡ The purpose of this note is to point out that if $f(x)$ is *analytic* in a sufficiently extended region of the complex plane the conditions relative to the vanishing of $\rho(x)$ can be greatly generalized. *It is sufficient that* $\rho(x)$ (supposed summable and non-negative over the interval of integration) *be different from zero over a set of positive measure*, or in other words that

$$\int_a^b \rho(x) dx > 0.$$

* Presented to the Society, September 9, 1930.

† See, for example, D. Jackson, *The Theory of Approximation*, New York, 1930, pp. 96–101.

‡ *A generalized problem in weighted approximation*, Transactions of this Society, vol. 26 (1924), pp. 133–154; see pp. 153–154. See also the paper by Shohat in the Mathematische Annalen cited below.