Les Problèmes des Isopérimètres et des Isépiphanes. By T. Bonnesen. Paris, Gauthier-Villars, 1929. iv+175 pp.

Here is the latest volume of the Collection Borel. Readers having as little Greek as the reviewer will appreciate the kindness of the author in translating the last word of the title on line 4 of the introduction; it means solids of given surface.

After introductory chapters on elementary maxima and minima, and Jensen's theory of convex functions, the author passes to the main subject matter, viz. the classical extremal properties of the circle and the sphere, and their generalizations, notably the Minkowski inequalities for the mixed volume of three convex solids. The very elegant methods of treatment are original with the author and consist in geometrical constructions, often surprisingly simple, on polygons and polyhedra, supplemented by elementary algebraic calculations and limit operations. The main difficulty in this subject consists in determining the cases where an inequality of the type considered reduces to an equality. This difficulty the author overcomes very neatly by setting up stronger inequalities, from which the answer may be read off at once. For instance, he replaces the ordinary isoperimetric inequality in the plane, $L^2/(4\pi)-S\geqq 0$, by $L^2/(4\pi)-S\geqq(\pi/4)(R-r)^2$, where L is the length of a convex curve, S the area it encloses, and R and r the radii of the circumscribed and inscribed circles. Now it is seen at once that in the ordinary inequality, equality takes place only when $R=r$, that is, for the circle.

The last chapter deals with the extension of the methods used to certain more general problems in the calculus of variations. There is also a fairly complete bibliography.

T. H. Gronwall

Géometrie sur les Surfaces et les Variétés Algébriques. By S. Lefschetz. Mémorial des Sciences Mathématiques, fascicule XL. Paris, Gauthier-Villars, 1929. 66 pp.

This pamphlet is a fairly complete account of that advanced part of the theory of algebraic surfaces and varieties which deals with the transcendental and geometric properties of a variety which are invariant under birational transformations. The timeliness of such a report is unquestionable. Since the appearance of Enriques' and Castelnuovo's article in the Encyclopedia (1914) the theory has been enriched by many new important results, and also a new development of the theory along the lines of analysis situs has taken place (Lefschetz). This development has thrown new light upon the stupendous progress already achieved by the theory thanks to the investigations of the Italian geometers Castelnuovo, Enriques, Severi, etc., and the French analysts Humbert, Picard, Poincaré. The pamphlet thus serves a useful purpose in furnishing a survey of the present state of the theory, and also gives the exact measure of the distance travelled by the theory since 1914.

The exposition centers around three main topics: the transcendental theory of the integrals attached to an algebraic variety V, the geometric theory of linear and continuous systems on V, and the analysis situs of V. Each of these topics leads in its own way to the very heart of the theory of algebraic varieties. The material is so arranged as to bring forth the far reaching con-