INVARIANT METHODS IN CLASSICAL DIFFERENTIAL GEOMETRY*

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There are two well known developments of classical differential geometry by invariant methods which are systematic and comprehensive, namely those of Cesàro† and Ricci.‡ Ricci's tensor analysis is so familiar as to need no detailed description, and the methods of Cesàro may perhaps best be set forth by comparing them with those of Ricci. In making this comparison there are two questions of major importance to consider, the one having to do with the point of departure of the theory, and the other with the method of differentiation.

Ricci based his developments on an analytic representation of the manifold under consideration. On the other hand, it was a matter of principle with Cesàro to get along without a fixed coordinate system to which to refer the manifold. Perhaps it is this strict adhesion to purely intrinsic geometry that has stood in the way of a more general adoption of Cesàro's methods. In any case, the opinion seems to be generally held that a more powerful theory is obtainable by combining the purely intrinsic methods with invariant methods based on representations by means of parameters or coordinates.

In contrast to Ricci's method of covariant differentiation, Cesàro employs the older method of intrinsic differentiation, differentiation along a curve with respect to the arc of the curve.§ It is probably the general opinion that here, too, Ricci's choice is the better one. This opinion is undoubtedly justifiable in the case of manifolds of higher dimensionality, but it is a question whether it can be upheld in the case of the manifolds of classical differential geometry.

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† *Lezioni di Geometria Intrinseca*, 1896.

‡ *Lezioni sulla Teoria delle Superficie*, 1898; also, Ricci and Levi-Civita, *Méthodes de calcul différentiel et absolu et leurs applications*, Mathematische Annalen, vol. 54 (1901), pp. 125-201.

§ The method was used earlier by various writers, notably, Lamé, Bonnet, Gilbert, and Enneper.