A CORRESPONDENCE BETWEEN IRREGULAR FIELDS*

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1. *Introduction.* Correspondences between fields are well known, and Dickson† has applied one to obtain a generalization of the theory of numbers. Here we give an instance of correspondence between irregular fields. An irregular field differs from a field only in the exclusion of an infinity of elements as divisors, instead of the uniquely excluded zero of a field. The postulates for an irregular field and numerous instances were given elsewhere.‡ The correspondence is established between the irregular field of all numerical functions and the irregular field of a certain infinity of power series with radius of convergence 1. For the series considered, addition and subtraction are interpreted as in the classical algebra of absolutely convergent series; multiplication and division receive wholly different interpretations. The simplest instance of the new multiplication is the process by which, when legitimate, a Lambert series is derived from a given power series.

It will be necessary for clearness to recall first a few definitions and theorems.

2. *The Irregular Field IF.* If $\xi(x)$ is uniform and defined for all integral values $n>0$ of x, $\xi(x)$ is called a *numerical function of* x. In what immediately follows, a relation involving n denotes the set of all relations obtainable from the given one by letting n range over all integers >0.

Let $\alpha(x)$, $\beta(x)$, $\cdots$, $\xi(x)$, $\eta(x)$, $\omega(x)$, $\cdots$ be the set of all numerical functions of x, the *unit function* $\eta(x)$ and the *zero function* $\omega(x)$ being the unique functions defined by

$$\eta(1) = 1, \ \eta(x) = 0, \ x \neq 1, \tag{1}$$

$$\omega(n) = 0. \tag{2}$$

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† This Bulletin, vol. 23 (1916), p. 109.

‡ Annals of Mathematics, vol. 27 (1926), p. 511; *Algebraic Arithmetic* (Colloquium Publications of the American Mathematical Society, vol. 7, 1927).