This theorem is a consequence of Theorems 1′ and 4′ and the result of Sierpinski, used by Professor Moore in the proof of Theorem 5.

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NOTE ON A SCHOLIUM OF BAYES

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In his fundamental paper on a posteriori probability,* Bayes considered a certain event M having an unknown probability p of its occurring in a single trial. In deriving his a posteriori formula he assumed that all values of p are equally likely, and he recommended this assumption for similar problems in which nothing is known concerning p. In the corollary to proposition 8 he derives the value

$$\int_0^1 \binom{n}{x} p^x(1-p)^{n-x}dp = \frac{1}{n+1}$$

for the probability of x successes in n trials. This result is independent of x; in a scholium he observes that this consequence is what is to be expected, on common sense grounds, from complete ignorance concerning p, and this concordance is considered to justify the assumption that all values of p are equally likely.†

In order to complete the argument of the scholium it is necessary to show that no other frequency distribution for p has the same property.

More precisely, given that a cumulative frequency function $f(p)$ has the property that for $0 \leqq x \leqq n$, x, n being integers,

$$\int_0^1 \binom{n}{x} p^x(1-p)^{n-x}df(p) = \frac{1}{n+1},$$

* Bayes, *An essay towards solving a problem in the doctrine of chances*, Philosophical Transactions of the Royal Society, vol. 53 (1763), pp. 370–418.

† In other words, the assumption "all values of p are equally likely" is *equivalent* to the assumption "any number x of successes in n trials is just as likely as any other number y, $x \leqq n$, $y \leqq n$." It has been suggested verbally by Mr. E. C. Molina that this proposition has a possible importance in certain statistical questions.