NOTE ON SEPARABILITY*

BY R. G. PUTNAM

The following theorems have been shown by R. L. Moore† to hold in a class D of Fréchet.‡

THEOREM 1. *In order that every subclass of a given class D of Fréchet should be separable, it is necessary and sufficient that every uncountable subclass of that class D should have a limit point.*

THEOREM 2. *If* D_s *is a separable class D, then every uncountable subclass of* D_s *contains a point of condensation.*

THEOREM 3. *Every subclass of a separable class D is itself separable.*

THEOREM 4. *In order that every uncountable subclass of a given class D should contain a point of condensation of itself, it is necessary and sufficient that every uncountable subclass of D should have a limit point.*

THEOREM 5. *In order that every ascending sequence of distinct closed subsets of a given class D should be countable, it is necessary and sufficient that every descending one should be.*

Theorems 3 and 4 follow from Theorems 1 and 2, and 5 is obtained with the aid of Theorems 1 and 4.

---

* Presented to the Society, September 6, 1928.

† Fundamenta Mathematicae, vol. 8, p. 189. Theorems 1, 2, and 3 have been previously considered by W. Gross in *Zur Theorie der Mengen, in denen ein Distanzbegriff definiert ist*, Sitzungsberichte, Wien, vol. 123 (1914), pp. 801–819. See also a reference to this article in *An acknowledgement*, by R. L. Moore, Fundamenta Mathematicae, vol. 8, p. 374.

‡ A class D of Fréchet is a class of elements which satisfy the following conditions:

1. With every pair of elements A and B there is associated a number $(A, B) = (B, A) \geqq 0$.
2. $(A, B) = 0$ if, and only if, $A = B$.
3. If A, B and C are any three elements, then $(A, C) \leqq (A, B) + (B, C)$.
4. The sequence of elements $P_1, P_2, P_3, \cdots$ converges to a limit P if and only if the distance (P, P_n) approaches zero as n becomes infinite. A class in which conditions 1, 2 and 4 hold but in which 3 need not hold is a class E.