# AN APPLICATION OF THE METHOD OF PARAMETERS TO LINEAR PARTIAL DIFFERENTIAL EQUATIONS*

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1. *Introduction.* The method of variation of parameters provides, as is well known, an elegant means of finding the complete primitive of a linear ordinary differential equation whose complementary function is given. In this paper it is shown that the method is applicable also to certain linear partial differential equations of the second order and that the results so obtained are interconnected with the familiar Laplace transformation.

In what follows we shall understand that the functions of $x$ and $y$ considered are of class $C''$ in a domain of definition $K$, where $C''$ is the class of functions which are continuous together with their first and second derivatives. We shall suppose that the complementary function of our differential equation is of the form $F(\alpha)+G(\beta)$, $\alpha$ and $\beta$ being known distinct expressions in $x$ and $y$, and $F$ and $G$ arbitrary functions, in $C''$. It may be readily verified that any differential equation having such a complementary function will be of the type

$$(1)\qquad (RD^2 + SDD' + TD'^2 + PD + QD')z = V,$$

where $R, S, \cdots, V$ are functions of $x$ and $y$ alone and $D\equiv \partial/\partial x$, $D'\equiv\partial/\partial y$. Not every equation conforming to (1), however, will have $F(\alpha)+G(\beta)$ as complementary function; a criterion that may be easily applied arises from the following conditions,† necessary and sufficient in class $C''$:

$$(2a)\qquad R\alpha_x^2 + S\alpha_x\alpha_y + T\alpha_y^2 = 0,$$

$$(2b)\qquad R\beta_x^2 + S\beta_x\beta_y + T\beta_y^2 = 0,$$

$$(2c)\qquad R\alpha_{xx} + S\alpha_{xy} + T\alpha_{yy} + P\alpha_x + Q\alpha_y = 0,$$

$$(2d)\qquad R\beta_{xx} + S\beta_{xy} + T\beta_{yy} + P\beta_x + Q\beta_y = 0.$$

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† It should be noticed that equations (2a)–(2d) place symmetrical hypotheses on $\alpha$ and $\beta$; this symmetry will be made use of later in the discussion.