ON THE TRANSFORMATION WHICH LEADS FROM THE BRIOSCHI QUINTIC TO A GENERAL PRINCIPAL QUINTIC*

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There are two well known methods of showing that any quintic equation (subject to certain restrictions which will not be gone into here) can be reduced to the important Brioschi normal form

$$w^5 - 10Zw^3 + 45Z^2w + k = 0, \tag{1}$$

with the aid of no irrationalities other than two square roots. Both methods employ a preliminary transformation to reduce the given quintic to the so-called principal form

$$Y^5 + 5aY^2 + 5bY + c = 0. \tag{2}$$

The coefficients of this transformation will involve one square root, in general, but no other irrationality. One method, that devised by Gordan † and later improved by Weber‡ and Dickson§, then sets up a transformation of the form $w = R(Y)$|| which leads from (2) to (1). This is, of course, the direct process, and the one which we should expect to follow, as a rule. However, in the present case, the required transformation is not at all simple, it is not easy to set up, and the constant term in the transformed equation is very difficult to determine explicitly.

Under the circumstances, it is by no means out of place to consider the second method, which, starting with equation(1), devises a transformation which leads from it to a principal quintic which can be identified with (2). It is then possible, by a known process, to set up a transformation leading from

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† Mathematische Annalen, vol. 28 (1887), pp. 152–166.

‡ *Algebra*, 2d ed., vol. I, 1898, pp. 263–267.

§ *Modern Algebraic Theories*, 1926, pp. 214–218.

|| $R(Y)$ is a rational function of Y, the coefficients of which are rational functions of the coefficients of (2) and the square root of the discriminant of (2).