

A LEMMA OF THE THEORY OF LINEAR
DIFFERENTIAL SYSTEMS*

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The reading of the preceding note by W. M. Whyburn suggests to me a lemma which, although it is simple, deserves to be stated and proved separately. The results of Whyburn† as well as those of Miss Whelan‡ may be derived immediately from this lemma.

We shall use the matrix notation and designate by German capitals the square matrices of n rows and n columns, and by small German letters the n -dimensional vectors, with the usual agreements as to the products of matrices by matrices or by vectors and so on.

By a solution of a linear differential system of n th order with integrable (L) coefficients we shall mean a vector whose components are absolutely continuous and satisfy the system almost everywhere.

LEMMA. *Let the coefficients $a'_{ij}(x)$, $a''_{ij}(x)$, $b'_i(x)$, $b''_i(x)$ of the linear differential system*

$$(1) \quad \frac{d\mathfrak{y}'(x)}{dx} = \mathfrak{A}'(x) \cdot \mathfrak{y}'(x) + \mathfrak{b}'(x),$$

$$(2) \quad \frac{d\mathfrak{y}''(x)}{dx} = \mathfrak{A}''(x) \cdot \mathfrak{y}''(x) + \mathfrak{b}''(x)$$

be integrable on $(0, 1)$. Let

$$\max_i \left\{ \sum_{j=1}^n \int_0^1 |a'_{ij}(x)| dx \right\} = M_1,$$

$$\max_i \left\{ \sum_{j=1}^n \int_0^1 |a''_{ij}(x)| dx \right\} = M_2,$$

* Presented to the Society, October 26, 1929.

† See also § 4 of Whyburn's paper referred to, on p. 94 of this issue.

‡ This Bulletin, vol. 35 (1929), pp. 112-119. Our lemma is but a slight modification of and is proved in a much the same way as lemma v (p. 116) of Miss Whelan.