ON RELATED DIFFERENCE AND DIFFERENTIAL SYSTEMS†

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In a recently published article‡ I considered the system of differential equations

$$(1)\qquad dy_i/dx = \sum_{j=1}^{j=m} A_{ij}(x)y_j + \beta_i(x), \qquad (i = 1, \cdots, m),$$

where $A_{ij}(x)$, $\beta_i(x)$, $(i, j=1, \cdots, m)$ are summable, real functions of the real variable x on $X: a \leqq x \leqq b$. I proved that for one set of definitions of the coefficients of the difference system

$$(2)\ \Delta^*y_i(r)/\Delta x(r) = \sum_{j=1}^{j=m} {}^*A_{ij}(r)^*y_j(r) + {}^*\beta_i(r), \ (i = 1, \cdots, m),$$

on E_n: $x_{0n}=a$, $x_{1n}, \cdots, x_{nn}=b$, where the asterisk indicates a function defined on E_n (replacing the bold-face type in my former paper), $^*f(r) = {}^*f(x_{rn})$ and $\Delta^*f(r) = {}^*f(r+1) - {}^*f(r)$, *every* solution of this system goes over in the limit as n, the number of points in E_n, becomes infinite, in such a way that X is *completely*§ subdivided to the corresponding solution, that is, the solution having the same initial values at $x=a$, of the differential system (1). The present paper shows that the conclusions of our former paper, stated above, are valid for *all possible* methods of defining the coefficients of system (2), so long as $\lim_{n\to\infty} {}^*A_{ij}(p) = A_{ij}(x)$, $\lim_{n\to\infty} {}^*\beta_i(p) = \beta_i(x)$, almost everywhere on X, and there exists a summable function $G(x)$ on X such that $|{}^*A_{ij}(p)|$, $|{}^*\beta_i(p)| < G(x)$ for all n, $(i, j=1, \cdots, m)$, on $I_{pn}: x_{pn} \leqq x \leqq x_{p+1,n}$, where p varies with n in such a way that the point x belongs to I_{pn}. It shows further that the approach to the limit is *uniform* on X and that all of these conclusions are valid for any law of complete subdivision of X by the points of E_n. Our former paper indicated ready adaptations of the work to non-

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§ By a *complete* subdivision is meant one such that lim $n\to\infty$ maximum $\Delta x(i) = 0$.