

Six Lectures on Recent Researches in the Theory of Fourier Series. By Ganesh Prasad. Calcutta, the University of Calcutta, 1928. xiv+139 pp.

This book gives an account of a series of lectures delivered at the University of Calcutta. It deals with recent developments in the theory of Fourier series, particularly those concerned with convergence, Cesàro summability, and properties of Fourier coefficients.

The first lecture is introductory in nature and outlines the various problems considered in the subsequent lectures. The second lecture deals with various criteria for the convergence of Fourier series, including those due to Lipschitz, Dini, Jordan, de la Vallée-Poussin, and Young. An account of Hardy's discussion of the logical relationship between these six criteria is then given and the lecture closes with a brief indication of Du Bois-Reymond's treatment of convergence in the case of discontinuities of the second kind and the consideration of two criteria for uniform convergence. The third lecture is concerned with examples of the failure of convergence and uniform convergence in the case of the Fourier series of continuous functions. It is based on the researches of Du Bois-Reymond, Schwarz, Fejér, Lebesgue, Steinhaus, and Neder.

The fourth lecture is devoted to the consideration of the Cesàro summability of Fourier series. It begins with the deduction of Fejér's criterion and Lebesgue's criterion for summability (C_1). Following this the author discusses certain types of discontinuities of the second kind where Lebesgue's criterion fails to apply. He seems to regard this failure as a serious defect, whereas it is merely the logical result of the fact that the condition is not a necessary as well as a sufficient one. The lecture continues with various other criteria for summability; one due to the author to replace that of Lebesgue in the instances mentioned above, Lebesgue's criterion for summability (C_2), Hardy and Littlewood's necessary and sufficient condition for Cesàro summability of some order, the Riesz-Chapman theorem regarding summability (C_k) where $k > 0$, and a few other related results.

The fifth lecture deals with the notion of "strong summability," introduced by Hardy and Littlewood in 1913. After giving the original Hardy-Littlewood theorem and a simplified proof of it due to Fejér, the author proceeds to consider certain generalizations due to Carleman, Sutton, and Hardy and Littlewood themselves. The lecture closes with a criticism of these various criteria, which is of the same nature as his criticism of Lebesgue's criterion, referred to above.

The sixth lecture deals first with various properties of Fourier coefficients, the discussion being based on the researches of Riemann, Parseval, F. Riesz, Fischer, and Neder. Following this the author takes up certain questions connected with the uniqueness of the representation of a function by a trigonometric series, and discusses various results of this type due to Cantor, Bernstein, Menchoff, Zygmund, Rajchman, and Nina Bary. The lecture closes with an account of some researches concerning the relation between the Fourier series and the Legendre series of a function, due to Fejér, Haar, and Lukacs. An appendix deals with certain aspects of the so-called "Riemannian theory of trigonometric series," the discussion centering about the work of Riemann and Zygmund. A second appendix contains notes, additions, and corrections.

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