

The Theory of Determinants, Matrices, and Invariants. By H. W. Turnbull. London and Glasgow, Blackie, 1928. 8 vo, vii + 338 pp.

Mechanical Features. This volume is attractively bound in smooth dark green cloth over extra heavy boards. It is printed from new and artistic type on special quality glazed paper and represents a real achievement in scientific printing on account both of the elegant arrangement of the formulas on the page and of the perfection of typography. The concise German symbolical notation for determinants and algebraic invariants, used consistently, afforded to the printers an opportunity to manufacture an exceptional mathematical book.

The Book as a Text Book. Scope. The reader of the work will do well to be reminded of the full title, as otherwise the impression might be gained that extensive study of the theory of determinants is necessary before invariant theory can be undertaken. This is hardly true literally.

We think that the author of this admirable book was not concerned primarily to produce a text-book. He probably was more interested in a scientific question; that of creating a logically developed exposition of the portion of invariant algebra which has its foundations largely in linearity and the linear properties of determinants and matrices. In carrying out this plan he was able to draw relevant material from a wide range of original sources, and has produced a work very likely to make a place for itself as a fundamental text. The book also is very stimulating reading for one who has read the original memoirs first.

In confining ourselves to linearity in invariant theory we necessarily omit practically all of reduction to fundamental systems, and syzygetic dependence. There is naturally, however, a chapter on Hilbert's Lemma on the basis of a system of polynomials, and the consequent proof of Gordan's theorem. A compensating advantage inherent in the plan is that most linear properties, such as, for example, the Laplace identities, are immediately extensible by generalization from two or three variables or sets to n variables, the generalization of the binary theory in those respects where generalization is feasible. The extension of all theories to the consequences of a literal number of variables is a characteristic feature of Professor Turnbull's work.

There are twenty-one chapters, with titles as follows: Matrices and determinants. Fundamental properties of the determinant. Linear properties (and) fundamental Laplace identities. Multiplication of matrices and determinants. Linear equation (and) corresponding matrices. Special types of determinant. Differentiation of a determinant. Binary forms. The general linear transformation. General properties of invariants. The first fundamental theorem. Multilinear forms. Symbolic methods of reduction. Seminvariants (and) algebraically complete systems. The Gordan-Hilbert finiteness theorem. Clebsch's theorem. Applications of Clebsch's theorem, canonical forms, etc. Invariant equations and Gram's theorem. Geometric interpretations. The general quadric. Miscellaneous recent developments.

Numerous and excellent lists of examples are interspersed and essential references are included.

The Aronhold-Clebsch symbolism is both so expressive and so concise and the author possesses such skill in all that pertains to elegance of notation that this book gives to casual reading the impression of being more elementary than