

KÖNIG AND KRAFFT'S ELLIPTIC FUNCTIONS

Elliptische Funktionen. By R. König and M. Krafft. Leipzig, Walter de Gruyter and Co., 1928. 263 pp.

Many are the methods of approach to the study of elliptic functions. Since Abel's classic memoir (1826), an outgrowth of his researches on the lemniscate definite integral, much attention has been given to the development of the theory of algebraic and of abelian functions and of the chapter in this theory which has to do with the subject of this book. Some, as Riemann (1857) and Weierstrass (1875-6) followed first the lead of Abel in the 1826 paper (anticipated by Gauss as early as 1798) but later redeveloped the theory using the algebraic differential equation of the first order as a cornerstone on which to build. Others, as Jacobi, (1829 and 1834), and Eisenstein, starting with properties of multiply periodic functions, have developed the theory of elliptic functions as an elegant chapter in the theory of functions. The geometric properties and applications interested Riemann, Clebsch, (1864), and Brill and Noether, (1874); while the latest and indeed the least traveled avenue of approach, viz., the number-theoretic, called also arithmetic, was employed by Dedekind and Weber, (1879), by Kronecker, (1881), and by Hensel and Landsberg, (1902).

This book by König and Krafft gives the first detailed exposition of the arithmetic theory as applied specifically to functions having the three fundamental properties which characterize elliptic functions, viz., (1) they form a linear manifold, (2) they form an algebraic field, (3) they are of genus one. The aim has been "not to set up theorems and formulas with lemmas and computations constructed for the purpose but to develop the results organically and genetically" from these three properties of which the first is the simplest and at the same time the most general. Thus it is evident that the scope of the book is considerably more than the development of the arithmetic theory of functions of genus one. As the introduction tells us: The reader becomes familiar with Riemann's doctrine, which lies at the bottom of the whole plan of the book, a doctrine which not only supplies a Riemann surface on which the function is single-valued, but which also enables the reader to understand the classification of elliptic functions as a member of a great organism, Riemann function-pairs as an instance of Riemann function-systems; moreover, according to Riemann and F. Klein, the fundamental law which rules the organism is a law of duality by which every theorem on functions can be restated for differentials. The reviewer thinks the book justified by their claim that "though the material is not new, the contribution is in the painstaking methodical simplifications made even in the things well known." It is a contribution which is welcome. The book can be read with pleasure by one who is familiar with the theory of elliptic functions in any of its phases or ramifications, and it can be used quite satisfactorily as a text with a class which knows the elements of function theory though not without considerable amplification of the