

their conciseness certain questions of emphasis. But as for the subject matter, there is no feeling of compression; on the contrary the author frequently pauses to suggest problems, explain difficulties and guiding principles, and emphasize values. For some theorems there are developed several distinct proofs, each bringing to light new aspects of the theory and new points of view. The essentials of the point set theory of separable spaces are developed at an early stage so that one may read the entire volume without outside reference. The material is well arranged and the printing is excellent, with exceptionally few errors. The book constitutes, in short, a notable presentation of an important chapter in modern mathematics.

P. A. SMITH

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### TONELLI ON TRIGONOMETRIC SERIES

*Serie Trigonometriche.* By Leonida Tonelli. Bologna, Nicola Zanichelli, 1928. viii+523 pp.

The extent of the existing literature on the theory of trigonometric series is tremendous and keeps increasing very rapidly. During the quarter of a century after 1900 the theory has made remarkable progress and it would not be an exaggeration to say that now it is of equal and fundamental importance for all branches of the modern mathematics including the theory of numbers on the one hand, and mathematical physics on the other. In spite of, or perhaps because of this, there is practically no place in the literature where an adequate account of the theory is given, except for the second volume of Hobson's *Theory of Functions* and two excellent but short reports by M. Plancherel (*L'Enseignement Mathématique*, vol. 24 (1925), and by E. Hilb and M. Riesz (*Encyklopädie der mathematischen Wissenschaften*, vol. II, 3<sub>2</sub>, 1924).

Under such circumstances the publication, by a mathematician of Tonelli's rank, of a large volume devoted exclusively to the theory of trigonometric series must be considered as a significant event, even if it does not represent a step toward the solution of the difficult problem of creating an all-inclusive treatise on trigonometric series.

The work under review originated as a course of lectures on trigonometric series delivered at the University of Bologna in 1924-1925 with a view "to expounding in a systematic manner the classical results, together with more recent investigations on these series." The author found it more convenient to abandon the usual order of treatment of trigonometric series. The book begins with a study of general trigonometric series, in order "to reveal at once the properties which are common to all such series," and subsequently passes to the discussion of the special properties of Fourier series. An advantage of such a treatment lies, according to the author's opinion, in the fact that "the theory of the general trigonometric series can be presented in a form essentially elementary in character, while the theory of Fourier series, for its complete development, requires speculations of a more advanced nature." No space is given to the "beautiful investiga-