NOTE ON LINEAR TRANSFORMATIONS OF n-ICS IN m VARIABLES*

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Let us consider the n-ic in m variables

$$F(x_1, x_2, \cdots, x_m) = 0. \tag{1}$$

If we subject (1) to the linear transformation

$$\begin{aligned} \rho x_1 &= a_{11}x_1' + a_{12}x_2' + a_{13}x_3' + \cdots + a_{1m}x_m', \\ \rho x_2 &= a_{21}x_1' + a_{22}x_2' + \cdots + a_{2m}x_m', \cdots, \\ \rho x_m &= a_{m1}x_1' + a_{m2}x_2' + a_{m3}x_3' + \cdots + a_{mm}x_m', \end{aligned} \tag{2}$$

we obtain

$$\begin{aligned} &F(a_{11}x_1' + a_{12}x_2' + a_{13}x_3' + \cdots + a_{1m}x_m', a_{21}x_1' + a_{22}x_2' \\ &\quad + a_{23}x_3' + \cdots + a_{2m}x_m', \cdots, a_{m1}x_1' + a_{m2}x_2' \\ &\quad + a_{m3}x_3' + \cdots + a_{mm}x_m') = 0. \end{aligned} \tag{3}$$

Note that in the expansion of (3) the coefficient of the term in $x_i'^n$, $(i=1, 2, 3, \cdots, m)$, is $F(a_{1i}, a_{2i}, a_{3i}, \cdots, a_{mi})$. A necessary and sufficient condition for this coefficient to vanish is that the point $P_i(a_{1i}, a_{2i}, \cdots, a_{mi})$ shall lie on the geometric locus of (1). To obtain the coefficient of such a term as $x_i'^r x_j'^{n-r}$ in the expansion of (3) we can put

$$\begin{aligned} &x_i' x_j' \neq 0, \; x_1' = x_2' = x_3' = \cdots = x_{i-1}' \\ &\quad = x_{i+1}' = x_{i+2}' = \cdots = x_{j-1}' = x_{j+1}' = \cdots = x_m' = 0, \end{aligned}$$

then use Taylor's Expansion on

$$\begin{aligned} &F(a_{1i}x_i' + a_{1j}x_j', a_{2i}x_i' + a_{2j}x_j', a_{3i}x_i' \\ &\quad + a_{3j}x_j', \cdots, a_{mi}x_i' + a_{mj}x_j') \equiv F(X_1 + X_1', \\ &X_2 + X_2', X_3 + X_3', \cdots, X_m + X_m'), \end{aligned} \tag{4}$$

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