ON THE INDEPENDENCE OF THE FIRST AND SECOND MATRICES OF AN ALGEBRA*

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1. *Introduction.* It is well known† that every linear associative algebra with a principal unit (modulus) is isomorphic with the algebra of its first matrices, and also with the algebra of its transposed second matrices. If the algebra has no principal unit, it can be represented as a matric algebra of $(n+1)$th order matrices.

The condition that the algebra have a principal unit is not, however, necessary in order that the algebra be isomorphic with the algebra of its first or second matrices, as can readily be seen from examples. In this paper necessary and sufficient conditions for this isomorphism are obtained.

2. *The Correspondence of Poincaré.* Consider a linear associative algebra $\mathfrak{A}$ over a field $\mathfrak{F}$ with n basal numbers $e_1, e_2, \cdots, e_n$, the constants of multiplication being c_{ijk}. Let us denote by R_i the matrix‡ (c_{isr}), and by S_i the matrix (c_{ris}), where r determines the row and s the column in which an element stands.

The conditions for associativity in $\mathfrak{A}$ may be written§

$$\sum_k c_{ikr}c_{jsk} = \sum_k c_{ijk}c_{ksr}, \quad (i,j,r,s = 1,2, \cdots, n). \tag{1}$$

If we form the matrices in which the respective members of the above equation stand in the rth row and sth column, we have

$$R_iR_j = \sum_k c_{ijk}R_k.$$

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† L. E. Dickson, *Algebras and their Arithmetics*, Chicago, 1923, p. 96.

‡ R_i and S_i are the first and transposed second matrices, respectively, of e_i. Dickson, loc. cit., p. 95.

§ Dickson, loc. cit., p. 92.