A THEOREM OF KUMMER'S CONCERNING THE SECOND FACTOR OF THE CLASS NUMBER OF A CYCLOTOMIC FIELD*

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Kummer† in a letter to Kronecker dated December 28, 1849, gave without proof the result that if

$$E_n = \epsilon^t\ ;\ t = 1 + sr^{-2n} + s^2r^{-4n} + \cdots + s^{(l-3)/2}r^{-(l-3)n},$$

and

$$\epsilon = \left(\frac{(1-\zeta^r)(1-\zeta^{-r})}{(1-\zeta)(1-\zeta^{-1})}\right)^{1/2}, \qquad \zeta = e^{2i\pi/l},$$

and

$$\prod_{i=1}^{(l-3)/2} E_i^{a_i} = \eta^l \tag{1}$$

where η is a unit in the cyclotomic field defined by ζ, then each $E_i^{a_i}$ is the lth power of a unit in the field, the a's being rational integers. Kummer refers to this result as follows:

"Sehr wichtiger Satz, welcher eine grosse Schwierigkeit hebt."

In the above statement the Kronecker-Hilbert‡ symbolic powers are employed; r is a primitive root of the odd prime l and s stands for the substitution (ζ/ζ^r). These symbolic powers have the following properties.§ If we denote by ω_k the integer in the field $k(\zeta)$ obtained from the integer $\omega=\omega_0$ by means of the substitution (ζ/ζ^{r^k}) then we write

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† Abhandlungen zur Geschichte der Mathematik, vol. 29 (1910), pp. 84–85.

‡ Hilbert, *Die Theorie der Algebraischen Zahlkörper*, Berichte der Deutschen Mathematiker-Vereinigung, 1894, p. 271.

§ Landau, *Vorlesungen über Zahlentheorie*, vol. 3, pp. 253–4, proves analogous properties for a field relative to k (ζ).