

By the use of Theorems III and IV, we derive  $(DM)$  from  $(L)$ . Let  $r_{in} - r'_{in}$  define a sequence of horizontal functions on  $I$ . The limit function is obviously a null function whose integral is zero. Conclusions 1 and 2 of  $(DM)$  follow immediately from  $(L)$ .

Conversely we derive  $(L)$  from  $(DM)$ . By Theorem IV,  $f(x)$  is bounded and integrable in  $I$ . Let  $h_n(x)$  be a set of horizontal functions on  $I$  with respect to  $I_{in}$  approaching  $f(x)$  as a limit function almost everywhere on  $I$ . Identify  $r_{in} = h_{in}$ . Also let  $H_n^{(k)}(x)$  be a horizontal function of index  $n$  on  $I$  associated with  $I_{in}$  and having  $f_k(x)$  as a limit function almost everywhere on  $I$ . Identify  $r'_{in} = H_{in}^{(n)}$  and Conclusion 2 of  $(L)$  follows from  $(DM)$ .

It is worthwhile emphasizing the role which horizontal functions, together with the principle isolated by R. L. Moore, are destined to play in the theory of functions of a real variable. They may be made the basis for a concise and elegant treatment of the greater part of the theory. The bulk of a book like Hobson's *Theory of Functions of a Real Variable*, volume I, may be reduced to one-third or less by their use. Schlesinger in the book under review has made a beginning in this direction.

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## FUBINI AND ČECH, PROJECTIVE DIFFERENTIAL GEOMETRY

*Geometria Proiettiva Differenziale*, Vol. I. By G. Fubini and E. Čech. Bologna, N. Zanichelli, 1926. 388 pp.

"Un nuovo indizio dei sentimenti fraterni che vanno sempre più legando fra loro i vari rami della matematica!"

These words of Segre were chosen by Wilczynski to be inscribed on the title page of his prize memoir *Sur la théorie générale des congruences*. Thus the founder of the American school of projective differential geometry indicated that he was studying the projective differential properties of a geometric configuration by the means of the invariants and covariants of a completely integrable system of linear homogeneous partial differential equations under a certain continuous group of transformations, in the sense of Lie. The same sentiment would be no less appropriate as a motto for the new book by Fubini and Čech, since these distinguished protagonists of the Italian school of projective differential geometry define a configuration by means of differential forms, after the manner of Gauss, and employ the absolute calculus of Ricci.

Those who know the absolute calculus only as it is used in the theory of relativity will be interested to see this geometric application of it. And those who know only Wilczynski's method of attacking a problem in projective differential geometry will be eager to learn this new theory. Wilczynski's method is particularly adapted to certain types of problems and has a power and elegance of its own. But it has some inconveniences. For instance, certain calculations become quite laborious, which are accomplished more easily and efficiently by the tensor analysis.

The first volume of the treatise before us is dedicated to the dean of differential geometers, Luigi Bianchi, and is devoted to the geometry of