

SCHLESINGER ON LEBESGUE INTEGRALS

Lebesguesche Integrale und Fouriersche Reihen. By L. Schlesinger and A. Plessner. Berlin and Leipzig, Walter de Gruyter Company, 1926. viii+229 pp.

This book is an outgrowth of a course of lectures by Professor L. Schlesinger at the University of Giessen in the winter semester of 1921-22 and was written with the cooperation of Dr. A. Plessner, who was at that time a student in the course, and later Professor Schlesinger's assistant. As the title indicates the subject is Lebesgue integrals and Fourier expansions from the point of view of Lebesgue's definition of integration.

Necessarily the first part is a study of the properties of real functions defined on point sets in a space of m dimensions. The first five chapters or 180 pages form a masterly exposition of the essential facts of real variable theory, particularly integration, and the sixth chapter of forty-two pages deals with Fourier series.

The headings give a sufficiently definite idea of the contents of each chapter. I. Fundamental concepts of points sets; II. Measure of point sets; III. Concerning functions of real variables; IV. The Lebesgue integral; V. Functions of one and two variables; VI. Fourier series.

Particular attention is directed to Chapter II in which the Borel-Lebesgue definition of measure is developed and compared with the older Peano-Jordan definition of content, and to Chapter IV where the Lebesgue integral is defined following Lebesgue's original geometric point of view. In the last section of this chapter (p. 129 ff.) attention is called to other useful definitions of the L -integral, due to Lebesgue himself, W. H. Young, J. Pierpont, and F. Riesz. This last definition will be discussed by the reviewer a little further on in greater detail, but at this point, it is well to note that mention should have been made along side of these of a two-way infinite series definition of a Lebesgue integral due to M. B. Porter.* By means of this definition the properties of the L -integral and the theorems of Lusin, Egoroff, and other important results are almost immediate.

Chapter VI contains an elegant and concise treatment of the most important newer results for Fourier series, notably those beginning with Lebesgue's *Leçons sur les Séries Trigonométriques*, 1906, which include the classic results of Riemann, Dini, Lipschitz, Dirichlet, Jordan, and others. The chapter closes with theorems on summability and convergence in the mean of Fejér, Cesàro, E. Fischer, and F. Riesz.

At the close of the preface is found a list of references to monographs, treatises, and encyclopedia articles whose content is closely related to the present text. Scattered throughout the book are to be found in footnotes, full references to the original memoirs bearing on all phases of the field.

* This Bulletin, vol. 28 (1922), pp. 105-8.