A CONNECTED AND CONNECTED IM KLEINEN POINT SET WHICH CONTAINS NO PERFECT SUBSET

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1. *Introduction.* Professor R. L. Moore has shown in this Bulletin (vol. 32, p. 331) that there exist point sets connected and connected im kleinen* which contain no arc. We shall prove in this paper the existence of such a set containing no *perfect* subset. The set has the additional property that it is contained in a *regular curve* (in the sense of K. Menger)†.

2. *The Sierpinski Regular Curve.* Let R be the Sierpinski regular curve,‡ defined as follows. Let T be an equilateral triangle. Divide T in 4 equal triangles. Let T_0, T_1, T_2 denote those three triangles which have a common vertex with T. Similarly divide each of the triangles T_0, T_1, T_2 in 4 equal triangles and let T_{00}, T_{01}, T_{02}, T_{10}, $\cdots$, T_{22} denote those having a common vertex with T_0 or T_1 or T_2; and so on *ad inf.*

The point set formed by the boundaries of all the triangles $T_{\alpha_1\alpha_2\cdots\alpha_k}(\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_i = 0, 1, \text{ or } 2)$ and all their limit points is the regular curve R.

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* A point set M is said to be *connected im kleinen* (or to be regular) if, for every point p and every positive number e, there exists a positive number d such that if x is any point of M at a distance from p less than d then x and p both lie in some connected subset of M of diameter less than e.

† A continuum C is called a regular curve if, for every positive number e, C can be expressed as the sum of a finite number of continua each of diameter less than e, each pair of continua having at most a finite number of points in common (see K. Menger, Mathematische Annalen, vol. 95 (1925), p. 300). Every regular curve is a continuous curve whose every subcontinuum is a continuous curve (see H. M. Gehman, Annals of Mathematics, vol. 27 (1925), p. 42).

‡ Prace Matematyczno-Fizyczne, vol. 27 (1915). The Sierpinski curve R contains three points of degree 2, a countable set of points of degree 4, the remainder being composed of points of degree 3. See also Comptes Rendus, vol. 160 (1915), p. 302.