

THE CAUCHY-HEAVISIDE EXPANSION FORMULA
AND THE BOLTZMANN-HOPKINSON
PRINCIPLE OF SUPERPOSITION*

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1. *Introduction.* It is the object of the present note to point out that the well known expansion theorem of Heaviside is an immediate corollary of the classical method given by Cauchy for finding a particular solution of a linear, non-homogeneous, ordinary differential equation when the general solution of the corresponding homogeneous equation is known. Various proofs of Heaviside's theorem have been given by Bromwich and Wagner, using contour integrals, by Carson and others and it seems hardly possible that the direct connection with Cauchy's method can have escaped attention. This method is interesting, furthermore, since it furnishes a rigorous proof of the useful Boltzmann-Hopkinson principle of superposition.† The explicit form of the expansion theorem in the case when the characteristic equation has multiple roots is also given.

2. *Cauchy's Method.* Although Cauchy's method is classical and to be found in the older books, it does not seem to find a place in the more modern texts and it will, therefore, be convenient to give an indication of its derivation. Let us suppose

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† The following citations may serve to give a proper orientation: Bromwich, T. J., *Normal coordinates in dynamical systems*, Proceedings of the London Society, vol. 15 (1916), pp. 401-448. Wagner, K. W., *Über eine Formel von Heaviside zur Berechnung von Einschaltvorgängen*, Archiv für Elektrotechnik, vol. 4 (1916), pp. 159-193. Carson, J. R., *On a general expansion theorem for the transient oscillations of a connected system*, Physical Review, vol. 10 (1917), pp. 217-225. Carson, J. R., *Theory of the transient oscillations of electrical networks and transmission lines*, Transactions A. I. E. E., vol. 38 (1919), pp. 345-427.