INTEGERS REPRESENTED BY POSITIVE TERNARY QUADRATIC FORMS*

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1. *Introduction.* Dirichlet† proved by the method of §2 the following two theorems:

THEOREM I. *$A=x^2+y^2+z^2$ represents exclusively all positive integers not of the form* $4^k(8n+7)$.

THEOREM II. *$B=x^2+y^2+3z^2$ represents every positive integer not divisible by* 3.

Without giving any details, he stated that like considerations applied to the representation of multiples of 3 by B. But the latter problem is much more difficult and no treatment has since been published; it is solved below by two methods.

Ramanujan‡ readily found all sets of positive integers a, b, c, d such that every positive integer can be expressed in the form $ax^2+by^2+cz^2+du^2$. He made use of the forms of numbers representable by

$$A, B, C = x^2+y^2+2z^2,$$
$$D = x^2+2y^2+2z^2,$$
$$E = x^2+2y^2+3z^2,$$
$$F = x^2+2y^2+4z^2,$$
$$G = x^2+2y^2+5z^2.$$

He gave no proofs for these forms and doubtless obtained his results empirically. We shall give a complete theory for these forms. These cases indicate clearly methods of procedure for any similar form.

For a new theorem on forms in n variables, see §9.

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† Journal für Mathematik, vol. 40 (1850), pp. 228–32; French translation Journal de Mathématiques, (2), vol. 4 (1859), pp. 233–40; Werke, vol. II, pp. 89–96.

‡ Proceedings of the Cambridge Philosophical Society, vol. 19 (1916–19), pp. 11–15. He overlooked the fact that $x^2+2y^2+5z^2+5u^2$ fails to represent 15.