$t=t_1s_1$. Since $(t_1s_1)^{n_1}=s_1^{n_1}$ is in (s), and n_1 is prime to the order of s_1, s_1 is a power of s. Thus t_1, as well as t, corresponds to t' in the isomorphism of G with $G/(s)$; but t_1 and t' are of the same order n_1. Every element of $G/(s)$ whose order is a divisor of m/m_1 corresponds to an element of G whose order is a divisor of m. It follows that t', and hence every element of $G/(s)$ whose order divides n, is commutative with every element whose order divides m/m_1. Hence $G/(s)$, being of order $<mn$, contains an invariant subgroup of order n. The corresponding subgroup of G, being of order $m_1n<mn$, also contains an invariant subgroup of order n.

Thus in all cases G contains a subgroup of order n. Similarly G contains a subgroup of order m. G is evidently the direct product of these two subgroups.

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A CUBIC CURVE CONNECTED WITH TWO TRIANGLES

BY H. BATEMAN

1. *Introduction.* If ABC, XYZ are two triangles, a cubic curve Γ_3 may be associated with them as follows.* Let (PQ, RS) denote the point of intersection of the lines PQ, RS; then Γ_3 is the locus of a point O such that (OA, YZ), (OB, ZX), (OC, XY) are collinear and also the locus of a point O for which (OX, BC), (OY, CA), (OZ, AB) are collinear. In fact when one set of three points is collinear the other set of three is also collinear. Take ABC as triangle of reference and let the points X, Y, Z have coordinates (x_1, x_2, x_3), (y_1, y_2, y_3), (z_1, z_2, z_3) respectively, then if (α, β, γ) are current coordinates

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* H. Grassmann, *Die lineale Ausdehnungslehre*, 1844, p. 226. The corresponding quartic surface connected with two tetrahedra is mentioned by H. Fritz, Pr. Ludw. Gymn. Darmstadt [reference taken from Jahrbuch der Fortschritte der Mathematik, vol. 21 (1889), p. 725] and by C. M. Jessop, *Quartic Surfaces*, Cambridge, 1916, p. 189.