ON A PROBLEM IN CLOSURE*

BY VIRGIL SNYDER

The following note concerns finite groups of birational transformations which leave an algebraic curve of genus 1 invariant.

Let the curve be expressed as a C_4 in S_3, intersection of two general quadric surfaces. This curve is invariant under the linear group G_8, of order eight, generated by the harmonic homologies defined by the self-conjugate tetrahedron associated with C_4. The points of C_4 are thus arranged in sets of 8, forming a linear I_8^1 of genus 0. If the curve be projected upon a plane from an arbitrary point, a plane quartic C_4 results with nodes at K_1, K_2. The four central homologies become four perspective quadratic involutions T_i with centers O_i not on C_4; the three axial involutions become three non-perspective quadratic inversions, with fundamental points O_{ik} for $T_iT_k = T_kT_i = T_{ki} = T_{lm}$ at the diagonal points of the quadrangle $O_1O_2O_3O_4$. The nodes K_1, K_2 are the other fundamental points for all seven operations.†

From the theorem of Bertini it follows that in any plane nodal quartic one and only one conic can be found meeting

* Presented to the Society, September 8, 1926.

† A brief synthetic outline of the properties of G_8 was first given by C. Segre, *Su una trasformazione irrazionale dello spazio . . .* , Giornale di Matematiche, vol. 21 (1883), pp. 355–378. This was amplified in connection with a larger problem by D. Montesano, *Su alcuni gruppi chiusi di trasformazioni involutorie nel piano e nello spazio*, Atti Istituto Veneto, (6), vol. 6 (1888), pp. 1425–1444. It is also contained in the papers by K. Meister, *Ueber die Systeme, welche durch Kegelschnitte mit einem gemeinsamen Polardreieck, bez. durch Flächen zweiten Grades mit einem gemeinsamen Polartetraeder gebildet werden*, Zeitschrift der Mathematik und Physik, vol. 31 (1886), pp. 321–347; vol. 34 (1889), pp. 6–24; 73–91 and by H. E. Timerding, *Ueber die quadratische Transformation durch welche die Ebenen des Raumes in ein System von Flächen zweiter Ordnung mit gemeinsamen Poltetraeder übergeführt werden*, Annali di Matematica, (3), vol. 1 (1898), pp. 95–117.