

## TWO-WAY CONTINUOUS CURVES\*

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A continuous curve  $M$  will be said to be a two-way continuous curve, or to be "two-way continuous," provided it is true that between every two points of  $M$  there exist in  $M$  at least two arcs neither of which is a subset of the other. A point  $P$  of a continuum  $M$  is a cut point of  $M$  provided it is true that the point set  $M - P$  is not connected. Every point of a continuum  $M$  which is not a cut point of  $M$  will be called a non-cut point of  $M$ .

In a paper *Concerning continua in the plane*,<sup>†</sup> among other results, I have established the following theorems which will be used in the proofs given in this paper.

I. *If  $K$  denotes the set of all the cut points of a continuum  $M$ , then every bounded, closed, and connected subset of  $K$  is a continuous curve which contains no simple closed curve.*

II. *Every cut point of the boundary of a complementary domain of a bounded continuum  $M$  is a cut point also of  $M$ .*

III. *If  $K$ ,  $H$ , and  $N$ , respectively, denote the set of all the cut points, end points,<sup>‡</sup> and simple closed curves of a continuous curve  $M$ , then  $K + H + N = M$ .*

IV. *If  $N$  denotes the point set consisting of all the simple closed curves contained in a continuous curve  $M$ , then every connected subset of  $M - N$  is arcwise connected.*

These results will be referred to by number as here listed. We shall now prove the following additional theorems.

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‡ For a definition of this term see R. L. Wilder, *Concerning continuous curves*, FUNDAMENTA MATHEMATICAE, vol. 7 (1925), p. 358.