

ON IRREDUNDANT SETS OF POSTULATES

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In a recent paper* Dr. H. M. Gehman has made the point that each of the irredundant sets of postulates proposed by the author in a previous paper† can be obtained from another set, which is not only not irredundant but not even independent, by the mechanical method given by the author and restated by Gehman in his paper.

This mechanical process converts any independent set of postulates into an irredundant set, and, as pointed out by Gehman, it has the same effect on certain non-independent sets. But, applied to an average set of postulates, this process yields postulates which are more or less complicated mixtures of irrelevant ideas. If an irredundant set of postulates is to be of any interest, the postulates of the set ought not to be such mixtures of irrelevant ideas; but the fact that the set can be obtained by the mechanical process from some other set is not an objection to it.

Indeed, any irredundant set can be considered as obtained by the mechanical method. For let A , B , C be irredundant. In view of the irredundance, B is equivalent to *if A then AB* , where AB means A and B . This equivalence is strict. It does not depend on the presence of other postulates. Similarly C is equivalent to *if A and AB then ABC* . Accordingly the set A , B , C can be restated in the form A , *if A then AB* , *if A and AB then ABC* . It is then clear that this set can be obtained by the mechanical method from the set A , AB , ABC . And the latter set is not even independent.

In this way, given any irredundant set, we can give it the form of a set obtained by the mechanical method if we

*This BULLETIN, vol. 32 (1926), pp. 159-161.

†TRANSACTIONS OF THIS SOCIETY, vol. 27 (1925), pp. 318-328.