RECENT PROGRESS WITH THE DIRICHLET PROBLEM*

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1. *The Classical Problem of Dirichlet, and its Status at the Beginning of the Twentieth Century.* Let T denote an open continuum in a euclidean space of n dimensions, and let t denote the set of its boundary points. A function, $F(p)$, defined on t, will be said to be continuous, if to every point, p, of t, and every $\epsilon > 0$, there corresponds a $\delta > 0$, such that $|F(p) - F(q)| < \epsilon$ for every point, q, of t, for which the distance $pq < \delta$. The classical problem of Dirichlet is then the following. *Given T and $F(p)$, to find a function continuous in $T+t$, reducing to $F(p)$ on t, and having in T continuous partial derivatives of second order which satisfy Laplace's differential equation*

$$\nabla^2 u = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \cdots + \frac{\partial^2 u}{\partial x_n^2} = 0.$$

We shall limit ourselves to the cases $n \leqq 3$.

We shall be concerned rather with the existence of the solution than with its actual construction. A domain, T, for which a solution exists corresponding to each continuous $F(p)$, will be called a *normal* domain.

In one dimension, the problem is always possible, for it amounts merely to finding a straight line through two points. In two and three dimensions, the most extensive results at the close of the last century were due to Poincaré and his method of *balayage*.† The contribution of Poin-

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† *Sur les équations aux dérivées partielles de la physique mathématique*, AMERICAN JOURNAL, vol. 12 (1890), pp. 211–294, and *Théorie du Potentiel Newtonien*, Paris, 1899. Carried out in two dimensions by A. Paraf, *Sur le problème de Dirichlet*, ANNALES DE TOULOUSE, vol. 6 (1892), pp. H. 1–H. 75.